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Control Chart Limits for Monitoring Process Mean Based on Downton's Estimator

Olatunde A. Adeoti,[a*†] John O. Olaomi[b] and Kayode S. Adekeye[c]

Control charts are important tools in statistical process control used to monitor shift in process mean and variance. This paper proposes a control chart for monitoring the process mean using the Downton estimator and provides table of constant factors for computing the control limits for sample size ($n \leq 10$). The derived control limits for process mean were compared with control limits based on range statistic. The performance of the proposed control charts was evaluated using the average run length for normal and non-normal process situations. The obtained results showed that the $\overline{X}_D$ control chart, using the Downton statistic, performed better than Shewhart $\overline{X}$ chart using range statistic for detection of small shift in the process mean when the process is non-normal and compares favourably well with Shewhart $\overline{X}$ chart that is normally distributed. Copyright © 2015 John Wiley & Sons, Ltd.

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1. Introduction

Control charts are important statistical process control tools for detecting an assignable cause of variation and monitoring of repetitive processes using either process mean or variability. The Shewhart $\overline{X}$ chart is an important, widely used statistical process control tool to monitor and control the process average, while the range (R) and standard deviation (S) are used to monitor the process variability. The Shewhart $\overline{X}$ and R or $\overline{X}$ and S charts are usually combined to evaluate the stability of a process. The R chart is used for sample sizes $n \leq 6$, and S chart is used for monitoring process variability for moderately large size, say $n \geq 10$.

A point beyond the conrol limits in the $\overline{X}$ chart and R or S chart indicates that the process is out of control, that is, the process is unstable and required necessary corrective action. (Montgomery[1]).

The control charts are usually based on the assumption that the distribution of the quality characteristics is normal or approximately normal. However, in practice, the assumptions are often violated, which makes the chart to be affected by outliers and sometimes sensitive to departure from normality. Specifically, the Shewhart $\overline{X}$ control chart is sensitive to outliers (Alwan[2]). When the normality assumption is violated, one approach is to transform the data to normality using the Box-Cox transformation method, which often leads to loss of information that may affect the conclusion drawn on such data.

The study of departure from normality and effect of outliers on Shewhart control chart for process mean and variability has been undertaken by many authors. Some authors have studied this by constructing control chart based on robust statistic estimation for process mean and variability (see Schiling and Nelson;[3] Yourstone and Zimmer;[4] Khoo;[5] Abu-Shawiesh;[6] Abbasi and Miller[7]).

Abu-Shawiesh[8] derived the control limits for the standard deviation control chart based on the mean absolute deviation (MAD) from the sample median. However, the derived control limits by Abu-Shawiesh[8] were for situation when a standard value of standard deviation is assumed known or specified by the management. The MAD control chart provides an alternative to the Shewhart S control chart for non-normal process. Adekeye *et al.*[9] extended the work of Abu-Shawiesh[8] deriving the control limits for process mean using MAD when quality characteristic under study is non-normal. The author concluded that the derived control limits are an improvement on the control limits of the Shewhart chart based on the standard deviation as a measure of controlling process variability. Abbasi and Miller[7,10] proposed a control chart called D chart to monitor process dispersion of a non-normal population based on the Downton estimator. They showed that the D chart is superior to R and close competitor to the S chart under the normality assumption but superior to both the R and S charts in detecting changes in process variability when the assumption of normality is violated. Abbasi and Miller[11] compared the performance of different dispersion charts under normally and non-normally distributed environments for different estimates of process standard deviation and concluded that under normally distributed

[a]Department of Statistics, University of South Africa and Department of Mathematics and Statistics, Bowen University, Iwo, Nigeria
[b]Department of Statistics, University of South Africa, Pretoria, South Africa
[c]Department of Mathematical Sciences, Redeemer's University, Ede, Nigeria
*Correspondence to: Olatunde Adeoti, Department of Statistics, University of South Africa and Department of Mathematics and Statistics, Bowen University, Iwo, Nigeria
[†]E-mail: tim_deot@yahoo.com

process, the S chart has the best overall detection ability for moderate to large sample size, and the performance of interquartile range chart for small sample size is also reasonable. They concluded that the D and mean deviation charts perform reasonably well for moderate to large sample sizes for the non-normal process environment with heavy-tailed symmetric and skewed distributions but are relatively affected if the sample size is small, while the R and S charts are extremely affected for almost all the non-normal processes.

In this paper, the concept of Abbasi and Miller[7,10], which is the development of Downton's statistic-based chart for process dispersion, is extended by developing the corresponding control chart limits for monitoring the process mean based on Downton's estimator.

2. Description of some estimators for process mean and variability

Different estimators for process mean and variability have been applied in the computation of control chart limits. Two of the estimators used in this paper will be discussed in this section stating its advantages and disadvantages. The estimators discussed are sample mean and the Downton estimator.

2.1. *Location estimator*

The sample mean: The sample mean, $\overline{X}$, is the most well known example of measure of averages and most efficient for normally distributed data (Rousseeuv[12]). It is the sum of all values divided by the number of the values. The sample mean, $\overline{X}$, is defined for random sample $X_1, X_2, \ldots \ldots X_n$ of size n as

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} \tag{1}$$

The sample mean is easy to compute, but it has zero breakdown point.

2.2. *Dispersion estimator*

Downton's estimator: The Downton estimator is a very robust dispersion estimator that is not affected by departure from normality (Abbasi and Miller[7]). The Downton estimator was first introduced by Downton[13] as an estimator for the standard deviation of a normal population, and it was shown by Barnett[14] that Downton's statistic is an unbiased estimator of σ. Let $X_1, X_2, \ldots \ldots X_n$ represent random sample of size n from a normal distribution with mean, μ, and standard deviation, σ, that is, $X \sim N(\mu, \sigma^2)$, and the corresponding order statistic be denoted by $X_{(1)}, X_{(2)}, \ldots \ldots X_{(n)}$ where $X_{(1)} \leq X_{(2)} \ldots \ldots . \leq X_{(n)}$. The Downton estimator is defined (see Abbasi and Miller[7]; Downton[13]; Abu-Shawiesh[15]) as

$$\begin{aligned} D &= \sqrt{\pi} \sum_{i=1}^{n} \frac{(2i-n-1)X_{(i)}}{n(n-1)} \\ &= \frac{2\sqrt{\pi}}{n(n-1)} \sum_{i=1}^{n} \left[i - \frac{1}{2}(n+1) \right] X_{(i)} \end{aligned} \tag{2}$$

The Downton statistic for process variability derived for sample size $n \leq 10$ is given in Table I.

Now, if $X_1, X_2, \ldots \ldots X_n$ is normally distributed, the unbiased estimator of D for σ based on the Downton estimator, that is, $E(D) = \sigma$ is given as

Table I. Downton's statistics for sample sizes ($\boldsymbol{n} \leq 10$)

Sample size (n)	Downton's statistics
2	$D = \frac{\sqrt{\pi}}{2}\left(X_{(2)} - X_{(1)}\right)$
3	$D = \frac{\sqrt{\pi}}{3}\left(X_{(3)} - X_{(1)}\right)$
4	$D = \frac{\sqrt{\pi}}{6}\left(\frac{3}{2}X_{(4)} + \frac{1}{2}X_{(3)} - \frac{1}{2}X_{(2)} - \frac{3}{2}X_{(1)}\right)$
5	$D = \frac{\sqrt{\pi}}{10}\left(2X_{(5)} + X_{(4)} - X_{(2)} - 2X_{(1)}\right)$
6	$D = \frac{\sqrt{\pi}}{15}\left(\frac{5}{2}X_{(6)} + \frac{3}{2}X_{(5)} + \frac{1}{2}X_{(4)} - \frac{1}{2}X_{(3)} - \frac{3}{2}X_{(2)} - \frac{5}{2}X_{(1)}\right)$
7	$D = \frac{\sqrt{\pi}}{21}\left(3X_{(7)} + 2X_{(6)} + X_{(5)} - X_{(3)} - 2X_{(2)} - 3X_{(1)}\right)$
8	$D = \frac{\sqrt{\pi}}{28}\left(\frac{7}{2}X_{(8)} + \frac{5}{2}X_{(7)} + \frac{3}{2}X_{(6)} + \frac{1}{2}X_{(5)} - \frac{1}{2}X_{(4)} - \frac{3}{2}X_{(2)} - \frac{5}{2}X_{(2)} - \frac{7}{2}X_{(1)}\right)$
9	$D = \frac{\sqrt{\pi}}{36}\left(4X_{(9)} + 3X_{(8)} + 2X_{(7)} + X_{(6)} - X_{(4)} - 2X_{(3)} - 3X_{(2)} - 4X_{(1)}\right)$
10	$D = \frac{\sqrt{\pi}}{45}\left(\frac{9}{2}X_{(10)} + \frac{7}{2}X_{(9)} + \frac{5}{2}X_{(8)} + \frac{3}{2}X_{(7)} + \frac{1}{2}X_{(6)} - \frac{1}{2}X_{(5)} - \frac{3}{2}X_{(4)} - \frac{5}{2}X_{(3)} - \frac{7}{2}X_{(2)} - \frac{9}{2}X_{(1)}\right)$

$$\widehat{\sigma} = \overline{D} \text{ and } \sigma_D = z_3\sigma \qquad \text{(Abbasi and Miller}^{7,10}\text{)}$$

where

$$z_3 = \frac{1}{\sqrt{n(n-1)}}\sqrt{n\left(\frac{1}{3}\pi + 2\sqrt{3} - 4\right) + \left(6 - 4\sqrt{3} + \frac{1}{3}\pi\right)} \tag{3}$$

The Downton estimator properties according to Abu-Shawiesh[15] can be summarized as follows:

- The Downton statistic does not require coefficients of $X_{(i)}$ or divisors like d_n.
- The asymptotic efficiency of the Downton estimator is 97.8%.
- The estimator is simple, unbiased and uncorrelated with the mean.
- The estimator is a linear combination of the ordered observations randomly and independently drawn from a normal population.
- It is not much affected by non-normality.
- It is slightly difficult to compute but high in efficiency, which makes it suitable for use.

3. Control chart based on Downton's estimator

The control chart based on Downton's statistic, hereby called D-control chart, is a dispersion control chart proposed by Abbasi and Miller[7,10] for monitoring the variability of a process. The D-control chart is a chart of subgroup standard deviations in which the control limits are set using an unbiased estimator based on the Downton statistic. It should be noted that the Downton estimator, D, is not much affected by non-normality (Abbasi and Miller[7]).

In the design of the D-chart, the relationship between D and σ is defined by a random variable Z where $Z = {}^{D}/_{\sigma}$ (Abbasi and Miller[10]). If $X_1, X_2, \ldots\ldots\ldots, X_n$ are normally distributed, then the unbiased estimator for σ and σ_D based on Downton's estimator is $\widehat{\sigma} = \overline{D}$ and $\sigma_D = z_3\sigma$, respectively, (Abbasi and Miller[10]) where $\overline{D} = \frac{\sum_{j=1}^{m} D_j}{m}$ is computed from an appropriate number of random samples obtained from a process during normal operating conditions, and z_3 is as defined in (3).

The three-sigma control limits for the estimate of the standard deviation based on the Downton statistic, as derived by Abbasi and Miller,[10] are as follows:

$$\begin{aligned} \text{LCL} &= max\left(0, \overline{D} - 3z_3\overline{D}\right) = Z_3\overline{D} \\ \text{CL} &= \overline{D} \\ \text{UCL} &= \overline{D} + 3z_3\overline{D} = Z_4\overline{D} \end{aligned} \tag{4}$$

where

$$Z_3 = 1 - 3z_3$$

and

$$Z_4 = 1 + 3z_3$$

4. Derivation of proposed control chart limits based on Downton's statistic

Our proposed chart denoted $\overline{X}_D$ for monitoring process mean is constructed based on the sample mean and the Downton estimator, which are used to estimate the process mean, μ, and process standard deviation, σ, respectively. Thus, let $X_{i1}, X_{i2}, X_{i3}, \ldots\ldots\ldots\ldots, X_{in}$ be a random sample of independent observations of size n taken over m subgroups, $i = 1,2,\ldots, m$. The sample sizes are assumed to be equal and taken from identical distribution function. To derive the control limits for the corresponding $\overline{X}_D$ control chart, the process mean, μ, is estimated by the average of the m subgroup sample means, and the process standard deviation is estimated using the average of the subgroup Downton's estimator values as an estimate of variability,that is, $\widehat{\sigma} = \overline{D}$, where

$$\overline{\overline{X}} = \frac{1}{m}\sum_{j=1}^{m}\overline{X}_j.$$

To obtain the control limits of the proposed chart based on the Downton statistic, we recall that the 3σ lower and upper control limits, LCL and UCL, of the Shewhart chart for the mean are given as

$$\begin{aligned} \text{LCL} &= \overline{X} - 3\sigma_{\overline{X}} \\ \text{CL} &= \overline{X} \\ \text{UCL} &= \overline{X} + 3\sigma_{\overline{X}} \end{aligned} \tag{5}$$

where $\overline{X}$ *is* the process average obtained from m subgroups consisting of n observations, and $\widehat{\sigma} = \overline{D}$ is the unbiased estimator of σ, $\overline{D} = \frac{\sum_{i=1}^{m} D}{m}$.

Therefore, the Shewhart control chart limits for proposed mean chart is derived as follows:

$$\begin{aligned} \text{LCL} &= \overline{X}_D - 3\frac{\overline{D}}{\sqrt{n}} = \overline{X}_D - A\overline{D} \\ \text{CL} &= \overline{X}_D \\ \text{UCL} &= \overline{X}_D + 3\frac{\overline{D}}{\sqrt{n}} = \overline{X} + A\overline{D} \end{aligned} \tag{6}$$

where $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}}$ *and* $\widehat{\sigma}_X = \overline{D}$, *and* $A = \frac{3}{\sqrt{n}}$

The values of z_3, Z_3 and Z_4 have been computed for different values of $n \leq 10$ and presented in Table II. For the range and standard deviation control chart, the corresponding table of factors for computing the control limits for the $\overline{X}$ control chart had been derived for monitoring process mean (see Montgomery[1]). The values of $\overline{X}_j$'s are plotted on the $\overline{X}_D$ chart for each sample period, if the value of $\overline{X}_j$, $j = 1, 2, \ldots, m$ falls outside the control limits in (6), the process is considered to be out-of-control. The summary of the control limits for the charts is given in Table III.

5. The performance of the $\overline{X}_D$ chart based on Downton's estimation

The Average run length (ARL) of the $\overline{X}_D$ based on the Downton estimator was studied and compared with the traditional $\overline{X}$ chart. The ARL represents the average number of points until chart signals where ARL $= \frac{1}{P_0}$ and P_0 is the probability that one point was out-of-control. It should be noted that in the literatures, the in-control average run length ARL_0 is 370.4 when the control limits are based

Table II. Control limit factors for constructing D chart

	Factors for control chart limits			
n	z_3	Z_3	Z_4	A
2	0.756	0	3.268	2.121
3	0.674	0	3.022	1.732
4	0.624	0	2.872	1.500
5	0.598	0	2.794	1.342
6	0.581	0	2.743	1.225
7	0.570	0	2.710	1.134
8	0.562	0	2.686	1.061
9	0.256	0.809	1.191	1.000
10	0.241	0.813	1.187	0.949

Table III. Summary of control limits for the proposed $\overline{X}_D$ and D and $\boldsymbol{\overline{X}}$ and R charts

$\overline{X}_D$ and D	$\boldsymbol{\overline{X}}$ and R
$\boldsymbol{\overline{X}}_D$chart	$\boldsymbol{\overline{X}}$ chart
LCL $= \boldsymbol{\overline{X}}_D - A\overline{D}$	LCL $= \overline{X} - A_2\overline{R}$
CL $= \overline{X}_D$	CL $= \overline{X}$
UCL $= \overline{X}_D + A\overline{D}$	UCL $= \overline{X} - A_2\overline{R}$
D chart	R chart
LCL $= Z_3\overline{D}$	LCL $= D_3\overline{R}$
CL $= \overline{D}$	CL $= \overline{R}$
UCL $= Z_4\overline{D}$	UCL $= D_4\overline{R}$

UCL, upper control limit; LCL, lower control limit; CL, center line.

on 3σ from the mean for the normally distributed data, and the out-of-control average run length ARL_1 for the two types of charts are then computed using MATLAB version 7.10 (MathWorks Inc, Natick, Massachusset, USA). Sample sizes of $n = 5$ are considered, and the process mean of observations is assume to have shifted from μ_0 to μ_1 where $\mu_1 = \mu_0 + k\sigma$ and $k \in \{1.00, 1.25, 1.50, \ldots\ldots, 2.00, 2.25, \ldots\ldots, 3.00\}$.The corresponding out-of-control ARL for the different possible shifts are displayed in Table IV. The out-of-control ARL performance of both charts for normally distributed data is the same when the magnitude of shift is moderate or large in the process observations and approximately the same when magnitude of shift is small, while for the non-normally distributed data, the results in Table IV indicate that the out-of-control signal is quickly detected by the mean chart based on Downton, $\overline{X}_D$ for small shifts when compared with the mean chart based on range statistic in the case of non-normality and for moderate and large shift, the $\overline{X}$ chart performs better than the $\overline{X}_D$ chart. The $\overline{X}$ control limits based on the Downton's estimator perform well for small shifts and can be used instead of the traditional $\overline{X}$ chart based on range statistic when normality assumption is violated, and the magnitude of shift is small.(k less or equal to 2).

6. Illustration

To illustrate the proposed control chart, data set taken from Montgomery[1] and a randomly generated data from the gamma distribution representing the normal and non-normal case are used.

Case 1 (Normal Process Data)

Data for normal process taken from Montgomery[1] (pg. 365), which is the glass container bursting-strength in 20 samples each of five observations to demonstrate the application of $\overline{X}_D$ chart based on Downton's estimator, is presented in Table V.

Case 2 (Non-normal Process Data)

Data for non-normal process are simulated data from *Gamma*(2, 1) distribution consisting of 20 samples each of size $n = 5$. The data and the summary statistics are presented in Table VI.

The summary information regarding the central line and the lower and upper control limits using the information in Table III for the Shewhart $\overline{X}$ and $\overline{X}_D$ are provided in Table VII for the two cases.

7. Simulation study

We simulate data for three distributions, namely normal, contaminated normal and non-normal for sample sizes $n = 5$ and $n = 10$ to evaluate the performance of the proposed chart. The normal distribution data are simulated with $\mu = 25$ and $\sigma = 5$, while the non-normal data are simulated gamma distribution with shape parameter, $\alpha = 2$, and scale parameter, $\beta = 1$. A (φ) 100% location-contaminated normal distribution data that contains (1-φ) 100% observations from $N(\mu_0, \sigma_0^2)$ and (φ) 100% observations from $(\mu_0 + \omega\sigma_0, \sigma_0^2)$, where $-\infty < \omega < \infty$ as defined in Nazir *et al.*[16] are generated. Here, the contaminated normal are based on 5% and 10% location-contaminated normal, that is, $\varphi = 0.05$ and $\varphi = 0.10$ and $\omega = 5$. The summary statistics, that is, mean, range and Downton values of the distribution for 1000 trials of 25 subgroup of size 5 and 10 are presented in Tables VIII and IX. Tables X and XI are the control limits of the $\overline{X}_D$and $\overline{X}$ for sample size $n = 5$ and 10.

Table IV. Average run length for $\overline{X}$ and $\overline{X}_D$ for varying shift values

Shift (k)	Normal		Non-normal	
	$\overline{X}$	$\overline{X}_D$	$\overline{X}$	$\overline{X}_D$
1.00	4.530	4.333	43.9	13.9
1.25	2.399	2.376	24.9	10.6
1.50	1.570	1.560	15.0	8.3
1.75	1.222	1.218	9.5	6.7
2.00	1.076	1.075	6.3	5.5
2.25	1.022	1.021	4.4	4.6
2.50	1.005	1.005	3.2	3.9
2.75	1.000	1.000	2.5	3.4
3.00	1.000	1.000	2.0	2.9

Table V. Data of glass container bursting-strength

Sample	Data					$\overline{X}$	R	D
1	265	205	263	307	220	252.0	102	44.12
2	268	260	234	299	215	255.2	84	35.79
3	197	286	274	243	231	246.2	89	39.16
4	267	281	265	214	318	269.0	104	39.69
5	346	317	242	258	276	287.8	104	47.31
6	300	208	187	264	271	246.0	113	51.21
7	280	242	260	321	228	266.2	93	39.69
8	250	299	258	267	293	273.4	49	23.57
9	265	254	281	294	223	263.4	71	29.95
10	260	308	235	283	277	272.6	73	29.95
11	200	235	246	328	296	261.0	128	56.17
12	276	264	269	235	290	266.8	55	21.62
13	221	176	248	263	231	227.8	87	35.62
14	334	280	265	272	283	286.8	69	26.40
15	265	262	271	245	301	268.8	56	21.44
16	280	274	253	287	258	270.4	34	15.95
17	261	248	260	274	337	276.0	89	34.02
18	250	278	254	274	275	266.2	28	13.64
19	278	250	265	270	298	272.2	48	19.31
20	257	210	280	269	251	253.4	70	28.00
Mean						**264.06**	**77.3**	**32.63**

Table VI. Simulated data from gamma distribution

Sample	Data					$\overline{X}$	R	D
1	5.3322	1.6282	2.7375	1.3331	2.9114	2.7885	3.9991	1.6447
2	3.0459	2.6155	1.3714	0.8135	0.1054	1.5903	2.9405	1.3614
3	0.4843	3.5481	0.5692	2.4122	2.1354	1.8298	3.0638	1.4124
4	2.1493	1.7685	1.8058	1.7103	2.3204	1.9509	0.6101	0.2837
5	9.4570	0.6028	6.5286	3.3771	1.9152	**4.3161**	**8.8542**	4.1680
6	1.5866	4.5316	1.9204	1.3096	3.0456	2.4788	3.2220	1.4004
7	1.4158	2.1936	0.2095	1.3508	2.3258	1.4991	2.1163	0.8994
8	4.4244	3.5653	0.2575	1.3244	1.2706	2.1684	4.1669	1.8834
9	4.2474	1.7091	0.7630	0.6952	3.4002	2.1630	3.5522	1.7262
10	0.5433	3.5296	1.0579	1.0684	3.8814	2.0161	3.3381	1.6210
11	4.7815	1.7801	1.0056	3.2471	1.5806	2.4790	3.7759	1.6335
12	3.3911	0.8803	2.8150	1.6409	1.4012	2.0257	2.5108	1.1404
13	1.3048	6.9152	1.4284	0.8340	1.6966	2.4358	6.0812	2.2246
14	0.8429	2.8310	0.8008	1.5005	2.9767	1.7904	2.1759	1.1234
15	0.5811	3.0972	0.6303	4.0908	2.3450	2.1489	3.5097	1.6810
16	0.8248	0.4327	1.8320	1.0160	2.6111	1.3222	2.1784	0.9505
17	4.2986	2.3797	0.5431	0.7942	0.6594	1.7350	3.7555	1.6358
18	0.8697	1.4262	2.9751	7.5170	2.0953	2.9767	6.6473	2.6303
19	1.3740	2.0722	0.6398	2.0961	2.4243	1.7213	1.7845	0.7604
20	2.1041	4.6715	1.3391	0.7804	0.7227	1.9235	3.9488	1.6340
Mean						**2.1851**	**3.6116**	**1.5907**

8. Discussion

The results of the derived control limits of the $\overline{X}_D$ and D charts for the normal process data in Table VII gave the same results as $\overline{X}$ and R chart, which is in conformity with the conclusion of Montgomery,[1] that is, the process is in statistical control. However, we find out that when the data are from a non-normal process, there was no point outside the control limits on the $\overline{X}_D$ chart (Figures 1 and 2), while a point was outside the limits on the range-based control chart for the simulated process data (Figures 3 and 4). Thus, we

Table VII. Control chart comparison between $\overline{X}$ and $\overline{X}_D$ and R and D

	Illustration 1				Illustration 2			
	$\overline{X}$	R	$\overline{X}_D$	D	$\overline{X}$	R	$\overline{X}_D$	D
UCL	308.66	164.49	307.85	91.17	4.2690	7.6385	4.3198	4.4444
CL	264.06	77.3	264.06	32.63	2.1851	3.6116	2.1851	1.5907
LCL	219.46	0	220.27	0	0.1012	0	0.0050	0

UCL, upper control limit; LCL, lower control limit; CL, center line.

Table VIII. Summary statistics for generated data $n=5$

	$n=5$			
Distribution	$\overline{X}$	$\overline{R}$	$\overline{X}_D$	$\overline{D}$
Normal	24.9436	11.6305	25.0729	4.9559
CN(0.05)	17.9978	18.4140	17.6101	8.2388
CN(0.10)	18.9169	20.5466	18.3412	8.9330
Gamma(2,1)	2.0314	3.1370	1.7892	1.3437

CN, contaminated normal.

Table IX. Summary statistics for generated data $n=10$

	$n=10$			
Distribution	$\overline{X}$	$\overline{R}$	$\overline{X}_D$	$\overline{D}$
Normal	24.9658	15.5585	24.9662	4.9422
CN(0.05)	18.1654	24.0819	17.7859	8.1736
CN(0.10)	18.7399	26.3503	18.1864	8.6866
Gamma(2,1)	2.0058	4.1457	1.7445	1.3307

CN, contaminated normal.

Table X. Control chart limits for generated data, $n=5$

	$n=5$					
	$\overline{X}$		$\overline{X}_D$		$\overline{X}$	$\overline{X}_D$
Distribution	LCL	UCL	LCL	UCL	CLI	CLI
Normal	18.2328	31.6544	17.1038	33.0420	13.4216	15.9382
CN(0.05)	7.3729	28.6227	4.3621	30.8581	21.2498	26.4960
CN(0.10)	7.0615	30.7723	3.9769	32.7055	23.7108	28.7286
Gamma(2,1)	0.2214	3.8414	−0.3715	3.9499	3.6200	4.3214

UCL, upper control limit; LCL, lower control limit; CN, contaminated normal.

Table XI. Control chart limits for generated data, $n=10$

	$n=10$					
	$\overline{X}$		$\overline{X}_D$		$\overline{X}$	$\overline{X}_D$
Distribution	LCL	UCL	LCL	UCL	CLI	CLI
Normal	20.1738	29.7578	19.0998	30.8326	9.5840	11.7328
CN(0.05)	10.7482	25.5826	8.0838	27.4880	14.8444	19.4042
CN(0.10)	10.6240	26.8558	7.8754	28.4974	16.2318	20.6220
Gamma(2,1)	0.7289	3.2827	0.1650	3.3240	2.5538	3.1590

UCL, upper control limit; LCL, lower control limit; CLI, control limit interval.

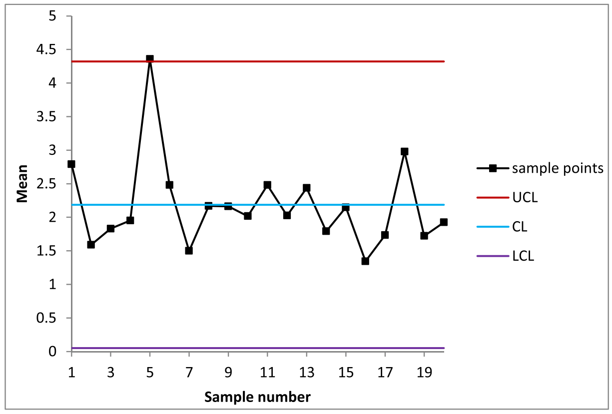

Figure 1. $\overline{X}_D$ control chart for non-normal simulated data

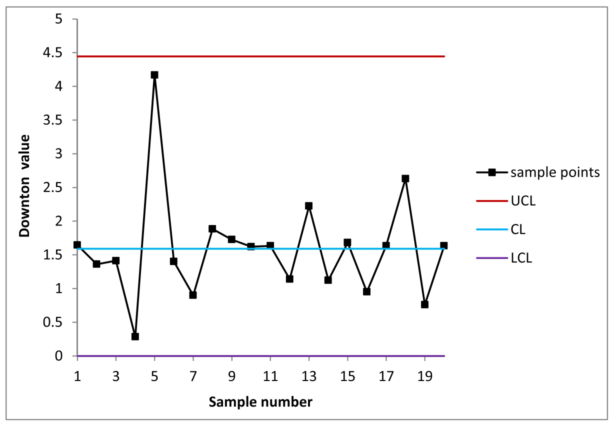

Figure 2. D-control chart for non-normal simulated data

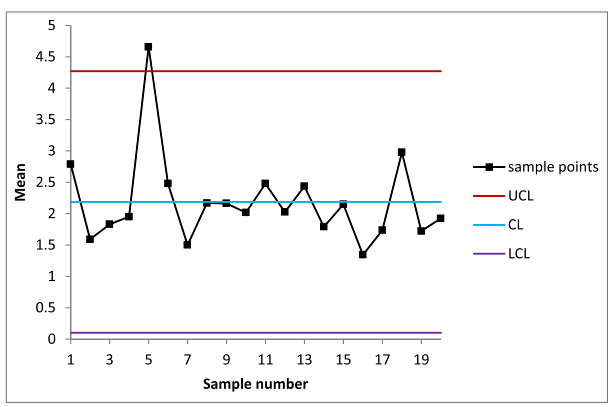

Figure 3. $\overline{X}$ control chart for non-normal simulated data

say that the process is in statistical control for the $\overline{X}_D$ and D charts because the stability of a process is determine from the mean and variability chart simultaneously. Hence, it is insensitive to departure from non-normality.

The result of simulation study in Tables X and XI shows that the control limit interval for the $\overline{X}_D$ is higher when Downton is used to compute control limits than when R chart was used for the normal and non-normal process. That is, the use of range-based control limits might lead to frequent false alarm in the process because there is the chance that some points will be outside the limits,

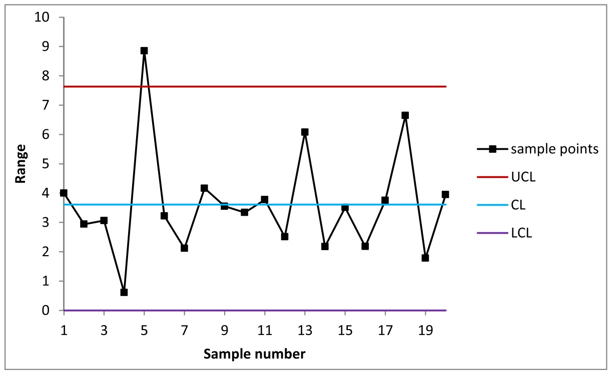

Figure 4. R control chart for non-normal simulated data

whereas when D is used to compute the limits, the points fall within the control limits for normal, contaminated normal and non-normal case. Also, the Downton-based control limits tend to accomodate more sample points than range-based control limits because there is no difference in the probability of not detecting a shift on the first subsequent sample for the proposed $\overline{X}_D$ and $\overline{X}$ control chart limits. Thus, we can say that $\overline{X}$ is sensitive to outliers. It was also observed that the Downton-based control limits detect changes in variability faster than R based control limits.

9. Conclusion

The control chart limits based on the Downton estimator, $\overline{X}_D$, were derived and have been proved to be more effective than the $\overline{X}$ chart limits based on the range statistic for monitoring process data when the normality assumption is violated. Therefore, it is recommended that the proposed $\overline{X}_D$ chart control limits based on Downton's statistic be used as an attractive alternative by quality control practitioners when the assumption of normality is violated.

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Authors' biographies

Olatunde Adeoti obtained his B.Sc (Hons) degree (First class honours) in Mathematics from the University of Ilorin, M.Sc (Statistics) from the University of Lagos and PhD in Statistics from the University of Ilorin, Ilorin, Nigeria. He is a member of the American Statistical Association and the Nigerian Statistical Association. His research interests include statistical process control, statistical quality control and management in healthcare and artificial neural network in statistical process control. He is currently a Senior Lecturer at the Department of Mathematics and Statistics, Bowen University, Nigeria.

John Olaomi obtained his B.Sc. (Hons) degree in Statistics and Postgraduate Diploma in Computer Science from the Obafemi Awolowo University, Ile-Ife, Nigeria. He obtained his M.Sc. and PhD (Statistics) from the University of Ibadan, Nigeria. He belongs to many professional societies, including the South Africa Statistical Association. His research interests include endogeneity (Econometrics), time series econometrics, statistical computing/consulting and operations research and sampling. He is currently an Associate Professor of Statistics at the University of South Africa.

Kayode Adekeye obtained his B.Sc, M.Sc (Statistics) from the University of Ilorin, Nigeria and Ph.D(Statistics) from the University of Dortmund, Germany. He is a Fellow of the Royal Statistical Society. His research interests include statistical quality control and industrial statistics, total quality management (TQM), design of experiment for quality improvement (with special emphasis on unreplicated experiments), exploratory data analysis, general and applied statistical methods. He is currently a Professor of Statistics, Department of Mathematical Sciences, Redeemer's University, Mowe, Nigeria.