

## RESEARCH ARTICLE

# Performance of median absolute deviation and some alternatives to median absolute deviation control charts for skewed and heavily tailed process

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## Abstract

In this paper,  $\bar{X}$  charts based on robust scale estimators (known as  $S_n$  and  $Q_n$  estimators) are proposed, and the performance of control charts based on median absolute deviation (MAD) is compared with those based on some alternatives to MAD, which do not need any location estimate, for normal, skewed, and heavily tailed distributions. MAD is often used as a substitute for standard deviation in constructing control charts due to its robustness. Three alternatives to MAD namely the  $S_n$ ,  $Q_n$ , and Downton ( $D$ ) are considered in this paper as location-free estimators. A simulation study was carried out to appraise the performance of the control charts based on the MAD,  $S_n$ ,  $Q_n$ , and  $D$  estimators. The average run length (ARL), median run length (MRL), standard deviation run length (SDRL), and control limits interval (CLI) were used to assess the performance of the four control charts. The results showed that MAD,  $S_n$ , and  $D$  are suitable estimators for standard deviation for mean charts while  $S_n$  and  $Q_n$  are suitable estimators for standard deviation for dispersion charts for skewed and heavily tailed distributions.

## KEYWORDS

alternative median absolute deviation, control charts, location-free estimators, median absolute deviation, skewed process data

## 1 | INTRODUCTION

Control chart is extensively used in statistical process control to discover when there is shift in a process. The basic assumption of control charts is that the quality characteristics are independently and identically distributed and follow normal distribution. The normality assumption is often violated by skewed and heavily tailed distributions, using traditional Shewhart control charts on such data might lead to inappropriate placement of control limits. A robust control chart is a good option in these kind of circumstances. Robust control charts estimate mean and variance by estimators that are insensitive to changes in the underlying distribution and resistant against the presence of outliers.<sup>25</sup> Hampel<sup>14</sup> introduced median absolute deviation (MAD) as an alternative robust estimate to sample standard deviation. The downsides of MAD come from the fact that it takes symmetric view on dispersion and has very low efficiency at Gaussian distributions.

Several authors have considered various robust measures to monitor process stability in statistical process control studies. Among these robust measures are trimmed mean of range,<sup>17</sup> inter quartile range (IQR),<sup>20,23</sup> Gini's mean difference,<sup>19</sup>

MAD,<sup>2,5,6</sup> dispersion on  $M$ -estimate,<sup>24</sup> the  $Q_n$  and  $S_n$  estimates,<sup>10,11</sup> and estimate obtained by the mean subgroup average deviation from the median deviation (MD).<sup>22</sup> Koukouvinos and Lappa<sup>16</sup> developed a moving average control chart using a robust scale estimator for process dispersion. Aslam et al.<sup>8</sup> developed control charts for monitoring process capability using MAD for some popular distributions, and Abu-Shawiesh et al.<sup>4</sup> proposed a robust control chart as an alternative to the Tukey's control chart based on the modified trimmed standard deviation.

This paper evaluates the performances of MAD and alternatives to median absolute deviation (AMAD)-based control charts for normal, skewed, and heavily tailed distributions. Das<sup>11</sup> did not present the  $\bar{X}$  chart's control limits based on  $S_n$  and  $Q_n$ . However, in practice, the control limits of process variability always rest on the process measure of location. Therefore, monitoring process variability without considering the process average leaves a gap to be filled. In this work, we extend Das's (2011) work to derive the corresponding control limits for the  $\bar{X}$  chart based on the  $S_n$  and  $Q_n$  estimators. The average run length (ARL), median run length (MRL), standard deviation run length (SDRL), and control limits interval (CLI) performance indices were used to determine the most appropriate estimator for skewed and heavy-tailed process.

## 2 | MAD- AND AMAD-BASED CONTROL CHARTS

### 2.1 | MAD-based control charts

The MAD was introduced by Hampel<sup>14</sup> as an alternative robust estimate to sample standard deviation. This estimator is very simple and easy to compute.

Let  $X_{ij}$  represent a random sample of size  $n$  taken over  $m$  subgroups,  $i = 1, 2, \dots, n$  and  $j = 1, 2, \dots, m$ ; the MAD is calculated as

$$\text{MAD} = 1.4826 \left[ \text{median}_i \left\{ \left| x_i - \text{median}_j (x_j) \right| \right\} \right]. \quad (1)$$

When MAD is used as an estimate of variability, then

$$\hat{\sigma} = b_n \overline{\text{MAD}}, \quad (2)$$

where  $\overline{\text{MAD}} = \frac{1}{m} \sum_{j=1}^m \text{MAD}_j$  is the median absolute deviation and  $b_n$  is a function of the sample size  $n$ .

$$\text{MAD}_j = \text{Median} \left| X_{ij} - \text{MD}_j \right|, \quad (3)$$

where  $\text{MD}_j = \text{Median } X_{ij}$ .

The control limits and center line for the Shewhart  $\bar{X}$  and MAD charts as modified by Adekeye<sup>6</sup> are as follows:

The MAD - control chart limits are

$$\begin{cases} \text{UCL}_s = B_4 b_n \overline{\text{MAD}} \\ \text{CL}_s = b_n \overline{\text{MAD}} \\ \text{LCL}_s = B_3 b_n \overline{\text{MAD}} \end{cases}, \quad (4)$$

and the corresponding control limits for the  $\bar{X}$  chart when the sigma is unknown are

$$\begin{cases} \text{UCL} = \bar{\bar{X}} + 3 \frac{b_n \overline{\text{MAD}}}{\sqrt{n}} \\ \text{CL} = \bar{\bar{X}} \\ \text{LCL} = \bar{\bar{X}} - 3 \frac{b_n \overline{\text{MAD}}}{\sqrt{n}} \end{cases}. \quad (5)$$

Let  $A_6 = 3 \frac{b_n}{\sqrt{n}}$ , then Equation (5) will be reduced to

$$\begin{cases} \text{UCL} = \bar{\bar{X}} + A_6 \overline{\text{MAD}} \\ \text{CL} = \bar{\bar{X}} \\ \text{LCL} = \bar{\bar{X}} - A_6 \overline{\text{MAD}} \end{cases} \quad (6)$$

The control chart constants ( $b_n$  and  $A_6$ ) values required for the calculation of the proposed control limits in Equations (4) and (6) can be found in Adekeye and Azubuiké.<sup>5</sup> Also, the constants  $B_3$  and  $B_4$  values can be found in Montgomery.<sup>18</sup>

## 2.2 | AMAD-based control charts

The three different alternative estimators to MAD considered in this work are the two robust scale estimators ( $S_n$  and  $Q_n$ ) by Rousseeuw and Croux<sup>21</sup> and Downton estimator by Downton.<sup>13</sup>

### 2.2.1 | S control chart based on $S_n$ estimator

Let  $X_i$ ;  $i = 1, 2, 3, \dots, n$  be a random sample of size  $n$  from a normal distribution with mean,  $\mu$ , and standard deviation,  $\sigma$ , and  $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ , where  $X_{(1)} \leq X_{(2)} \leq \dots \leq X_{(n)}$  be the corresponding order statistic. The estimator  $S_n$  is defined by

$$S_n = 1.1926 \left[ \text{median}_i \left\{ \text{median}_j |x_i - x_j| \right\} \right]; i \neq j. \quad (7)$$

Das<sup>11</sup> introduced  $S_n$  estimator into control charting procedure. Thus, the control limits for the Shewhart-S control charts based on  $S_n$  are calculated using

$$\begin{cases} \text{UCL}_S = c_4 c_n \bar{S}_n + 3 c_n \bar{S}_n \sqrt{1 - c_4^2} = B_7 \bar{S}_n \\ \text{CL}_S = c_4 c_n \bar{S}_n = c_4^* \bar{S}_n \\ \text{LCL}_S = c_4 c_n \bar{S}_n - 3 c_n \bar{S}_n \sqrt{1 - c_4^2} = B_8 \bar{S}_n \end{cases} \quad (8)$$

The values of  $c_4^*$ ,  $B_7$ , and  $B_8$  are given in Das.<sup>11</sup>

### 2.2.2 | S control chart based on $Q_n$ estimator

The  $Q_n$  estimator shares the same properties with the  $S_n$  estimator. It has a simple and unambiguous formula, which is appropriate for asymmetric distribution. The estimator  $Q_n$  is a location-free estimator and is defined by

$$Q_n = 2.2219 \left[ 0.25^{\text{th}} \text{ quantile} \left\{ |x_i - x_j|_{(k)}; i < j \right\} \right], \quad (9)$$

where  $k = \binom{h}{2} \approx \binom{n}{2} / 4$ ,  $h = \frac{n}{2} + 1$ , and  $|x_i - x_j|_{(k)}$  is the  $k$ th order statistic of the  $\binom{n}{2}$  interpoint distance  $|x_i - x_j|$ .

The control limits for the Shewhart-S control charts based on  $Q_n$  according to Das<sup>11</sup> are

$$\begin{cases} \text{UCL}_S = c_4 d_n \bar{Q}_n + 3 d_n \bar{Q}_n \sqrt{1 - c_4^2} = B_9 \bar{Q}_n \\ \text{CL}_S = c_4 d_n \bar{Q}_n = c_4^{**} \bar{Q}_n \\ \text{LCL}_S = c_4 d_n \bar{Q}_n - 3 d_n \bar{Q}_n \sqrt{1 - c_4^2} = B_{10} \bar{Q}_n \end{cases} \quad (10)$$

The values of the constants  $d_n$ ,  $c_4^{**}$ ,  $B_9$ , and  $B_{10}$  are available in Das.<sup>11</sup>

### 2.2.3 | Downton's estimator

The Downton's estimator was introduced by Downton<sup>13</sup> as an estimator for standard deviation of a normal population. Downton's statistic is an unbiased estimator of  $\sigma^9$  and a very robust dispersion estimator that is not affected by departure from normality.<sup>1,7</sup> The Downton's statistic does not require coefficients of  $X_{(i)}$  or divisors like  $d_n$  and its asymptotic efficiency is 97.8% (Abu-Shawiesh, 2000).<sup>3</sup>

The Downton's estimator is

$$D = \frac{2\sqrt{\pi}}{n(n-1)} \sum_{i=1}^n \left( i - \frac{1}{2}(n+1) \right) X_{(i)}. \quad (11)$$

Abu-Shawiesh and Abdullah<sup>3</sup> presented the control limits for mean and dispersion control chart based on the Downton's estimator.

## 2.3 | Control limits for proposed $\bar{X}$ chart based on $S_n$ and $Q_n$

The control limits for the  $\bar{X}$  chart based on  $S_n$  and  $Q_n$  was not provided by Das.<sup>11</sup> In this work, we extend the work of Das<sup>11</sup> to derive the corresponding control limits for the  $\bar{X}$  chart based on  $S_n$  and  $Q_n$  estimators.

Using the 3-sigma limits approach, the corresponding control limits for the  $\bar{X}$  chart was derived by modifying the mean chart based on  $S_n$  (Equation 7) and  $Q_n$  (Equation 9).

Traditionally, the 3-sigma limits for the  $\bar{X}$  control chart is

$$\begin{cases} \text{UCL} = \bar{\bar{X}} + 3 \frac{\sigma_{\bar{X}}}{\sqrt{n}} \\ \text{CL} = \bar{\bar{X}} \\ \text{LCL} = \bar{\bar{X}} - 3 \frac{\sigma_{\bar{X}}}{\sqrt{n}} \end{cases}. \quad (12)$$

If the estimate of  $\sigma_{\bar{X}} = c_n \bar{S}_n$ , then the  $\bar{X}$  control chart limits based on  $S_n$  will be

$$\begin{cases} \text{UCL} = \bar{\bar{X}} + 3 \frac{c_n \bar{S}_n}{\sqrt{n}} \\ \text{CL} = \bar{\bar{X}} \\ \text{LCL} = \bar{\bar{X}} - 3 \frac{c_n \bar{S}_n}{\sqrt{n}} \end{cases}. \quad (13)$$

Let  $A_7 = 3 \frac{c_n}{\sqrt{n}}$ , then Equation (13) will become

$$\begin{cases} \text{UCL} = \bar{\bar{X}} + A_7 \bar{S}_n \\ \text{CL} = \bar{\bar{X}} \\ \text{LCL} = \bar{\bar{X}} - A_7 \bar{S}_n \end{cases}. \quad (14)$$

Similarly, if the estimate of  $\sigma_{\bar{X}} = d_n \bar{Q}_n$ , then,  $3\sigma$  control limits for the  $\bar{X}$  chart in Equation (12) will become

$$\begin{cases} \text{UCL} = \bar{\bar{X}} + 3 \frac{d_n \bar{Q}_n}{\sqrt{n}} \\ \text{CL} = \bar{\bar{X}} \\ \text{LCL} = \bar{\bar{X}} - 3 \frac{d_n \bar{Q}_n}{\sqrt{n}} \end{cases} \quad (15)$$

Let  $A_8 = 3 \frac{d_n}{\sqrt{n}}$  and  $d_n$  is as earlier defined, then Equation (15) will be reduced to

$$\begin{cases} \text{UCL} = \bar{\bar{X}} + A_8 \bar{Q}_n \\ \text{CL} = \bar{\bar{X}} \\ \text{LCL} = \bar{\bar{X}} - A_8 \bar{Q}_n \end{cases} \quad (16)$$

Therefore, the expression derived in Equation (14) is the  $3\sigma$  control limits for the  $\bar{X}$  chart based on the  $S_n$  estimator, while Equation (16) is the derived  $3\sigma$  control limits for the  $\bar{X}$  chart based on the  $Q_n$  estimator.

## 2.4 | Performance evaluation

To measure the performance of control charts based on the three alternative MAD estimators considered in this study, the following indices were used: ARL, MRL, SDRL, and CLI. The computation formulas of the indices are as given in Abu-Shawiesh et al.<sup>4</sup>

## 3 | RESULTS

### 3.1 | Simulation study

In this section, the performances of the proposed control charts are compared under different distributions. Five thousand iterations of Monte Carlo simulation of five sample sizes for 30 subgroups were generated from Normal, Weibull, Chi-square, Contaminated Normal, and Mixture model (Normal and  $t$ ) distributions to evaluate the performance of different competing methods. The models' specifications are as follows:

**Model (1):** Normal distribution  $N(0, 1)$

**Model (2):** Weibull (1.2, 5.5)

**Model (3):** Chi-Square (7.2, 3)

**Model (4):** Contaminated Normal:  $0.3 \cdot \text{norm}(0, 1) + 0.7 \cdot \text{norm}(5, 15.5)$

**Model (5):** Mixture model:  $0.7 \cdot \text{norm}(0, 1) + 0.3 \cdot t(5, 20)$

The mean, MAD (Equation 3),  $S_n$  (Equation 7),  $Q_n$  (Equation 9), and  $D$  (Equation 11) for the generated data were computed and employed in the derived and existing control limit formulas. The performance indices (PI) were computed for the four control charts considered in this study and the obtained results are presented in Tables 1 and 2 for the five generated datasets.

It should be noted that the data generated with Model 1 is used in this work as a control, because the robust estimators are meant for skewed and heavy-tailed distribution.

TABLE 1 Performance indices for  $\bar{X}$  charts based on different scale estimators

Model	Performance indices	Control chart based on			
		MAD	$S_n$	$Q_n$	$D$
Model 1	CLI	2.6909	2.7166	3.8058	2.7201
	ARL	29	29	30	29
	MRL	29	29	30	29
	SDRL	0.0	0.0	0.0	0.0
Model 2	CLI	8.8263	8.7799	12.419	9.5912
	ARL	30	30	30	30
	MRL	30	30	30	30
	SDRL	0.0	0.0	0.0	0.0
Model 3	CLI	10.7401	10.5193	16.2304	12.3672
	ARL	30	30	30	30
	MRL	30	30	30	30
	SDRL	0.0	0.0	0.0	0.0
Model 4	CLI	33.1276	32.3168	48.9785	29.3328
	ARL	30	30	30	30
	MRL	30	30	30	30
	SDRL	0.0	0.0	0.0	0.0
Model 5	CLI	7.0125	6.7272	10.4054	7.4105
	ARL	29	29	30	29
	MRL	29	29	30	29
	SDRL	0.0	0.0	0.0	0.0

TABLE 2 Performance indices for  $S$  chart based on different scale estimators

Model	Performance indices	Control chart based on			
		MAD	$S_n$	$Q_n$	$D$
Model 1	CLI	2.0950	1.9879	1.8895	1.9904
	ARL	30	30	30	30
	MRL	30	30	30	30
	SDRL	0.0	0.0	0.0	0.0
Model 2	CLI	6.8716	6.4248	6.1658	7.0183
	ARL	29	29	29	29
	MRL	29	29	29	29
	SDRL	0.0	0.0	0.0	0.0
Model 3	CLI	8.3615	7.6976	8.0579	9.0496
	ARL	29	29	29	29
	MRL	29	29	29	29
	SDRL	0.0	0.0	0.0	0.0
Model 4	CLI	25.7909	23.6482	24.3163	21.4641
	ARL	30	30	30	30
	MRL	30	30	30	30
	SDRL	0.0	0.0	0.0	0.0
Model 5	CLI	5.4595	4.9227	5.1659	5.4226
	ARL	28	28	28	28
	MRL	28	28	28	28
	SDRL	0.0	0.0	0.0	0.0

FIGURE 1 Histogram of gas turbine cycle times

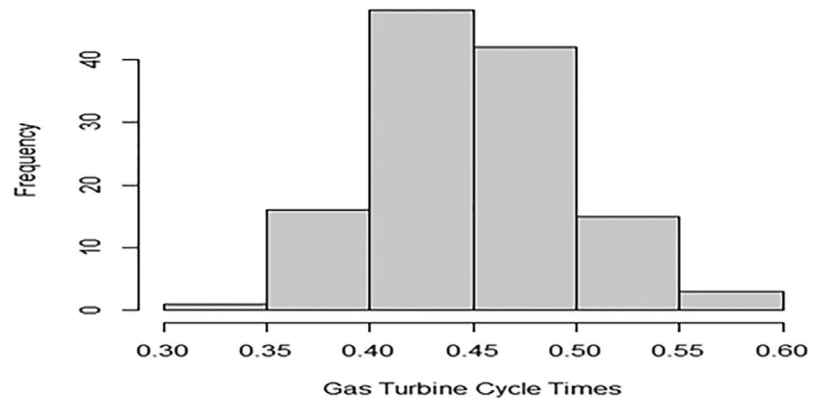
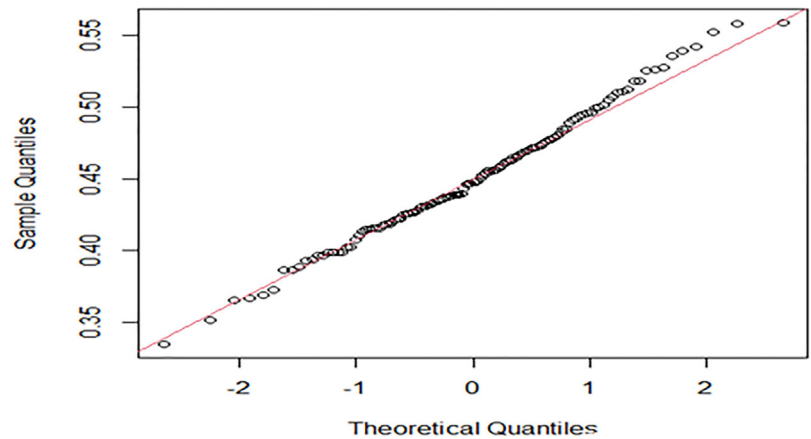


FIGURE 2 Q-Q plot of gas turbine cycle times



### 3.2 | Real-life data: Gas turbine cycle time

Real-life data of 125 gas turbine cycle times from Kenett and Zacks<sup>15</sup> was used to illustrate the proposed methods. The histogram (see Figure 1) and QQ plot (see Figure 2) of the data reveal that the gas turbine cycle times data are not from a normal distribution. The skewness was also found to be 0.1721, which confirmed that the gas turbine cycle times data are positively skewed, and thus a good fit for this study.

To be able to determine the control limits derived in Section 2, the 125 gas turbine cycle times data were divided into 25 subgroups of five observations, and the  $(\bar{X})$ ,  $S$ ,  $MAD$ ,  $S_n$ ,  $Q_n$ , and  $D$  were computed for the 25 subgroups. The four described robust estimators in Section 2 were computed and used to construct  $\bar{X}$  and  $S$  control charts based on  $MAD$ ,  $S_n$ ,  $Q_n$ , and  $D$  estimators.

Figures 3 and 4 represent the  $\bar{X}$  and  $S$  control charts, respectively, for the data using the four estimators under consideration. The control limits and the CLI for  $\bar{X}$  and  $S$  charts are presented in Tables 3 and 4, respectively.

TABLE 3  $\bar{X}$  charts control limits and CLI based on scale estimators for gas turbine data

Limits	MAD	$S_n$	$Q_n$	$D$
UCL	0.5078	0.5099	0.5381	0.5112
CL	0.4508	0.4508	0.4508	0.4508
LCL	0.3939	0.3918	0.3636	0.3905
CLI	0.1139	0.1182	0.1746	0.1207

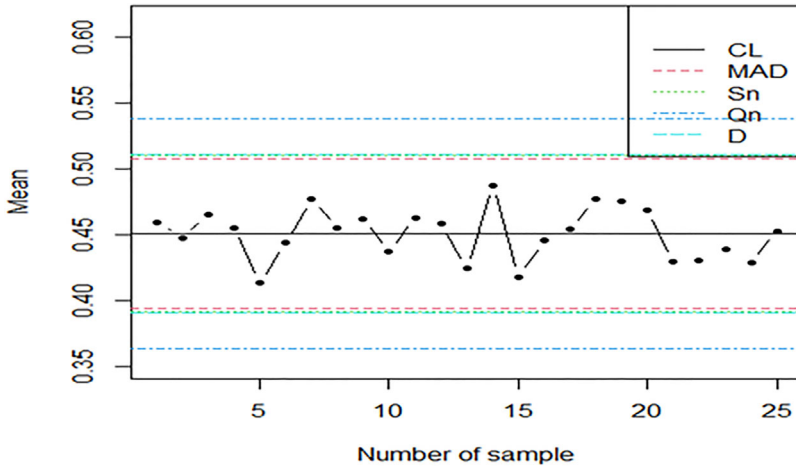


FIGURE 3 X-bar control chart based on scale estimators

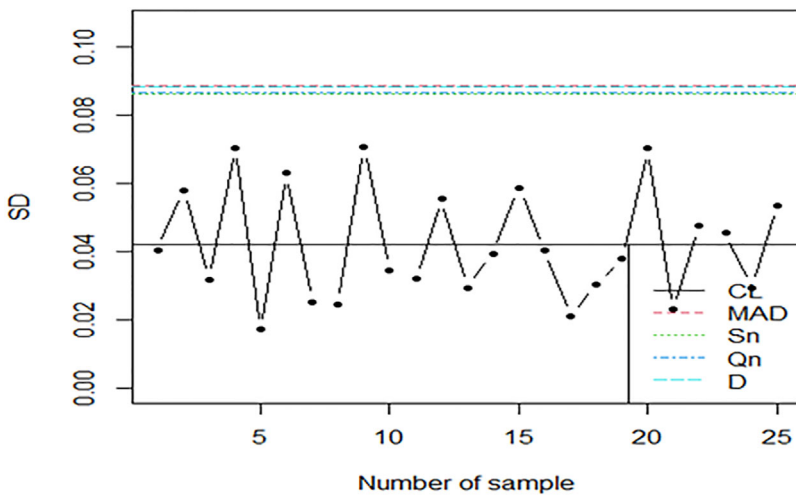


FIGURE 4 Dispersion control chart based on scale estimators

## 4 | DISCUSSION

Based on the results in Table 1, it is clear that for the  $\bar{X}$  control chart, all other estimators except  $Q_n$  estimator have approximately the same CLI. Furthermore, the MAD based  $\bar{X}$  control chart has the smallest CLI for all the models followed by  $S_n$  and  $D$  estimators. Hence, MAD,  $S_n$ , and  $D$  are suitable estimators for standard deviation in constructing mean charts for skewed and heavily tailed distributions, but MAD would detect out-of-control faster than  $S_n$  and  $D$  estimators. The ARL, MRL, and SDRL have the same results for all the models considered as none of the models detect out-of-control points. The ARL values in Tables 1 and 2 were obtained from the generated values using the distribution parameters under study. Thus, the values of ARLs are out-of-control, ARL that should be small.

When comparing the performance indices of the five models, it was observed that Model 4 had the highest control limit interval, which implies that there is a tendency to skip some out-of-control points for Model 4. It should be noted that the CLI is synonymous with the confidence limit of a parameter. Thus, the smaller the CLI, the greater the chance of the control chart in detecting out-of-control points.

TABLE 4 S chart control limits and CLI based on scale estimators for gas turbine data

Limits	MAD	$S_n$	$Q_n$	$D$
UCL	0.0887	0.0865	0.0867	0.0883
CL	0.0425	0.0414	0.0415	0.0428
LCL	0.0000	0.0000	0.0000	0.0000
CLI	0.0887	0.0865	0.0867	0.0883

The ARL and MRL for Models 2, 3, and 4 are the same, while those of Models 1 and 5 are also the same. It should however be noted that the difference between the values of PI are not significantly different. The implication of the results is that there is not much difference between the performance of the control charts based on MAD and AMAD considered in this study.

The results on the real-life data displayed in Tables 3 and 4 corroborate the results from generated data. The control limits of the  $\bar{X}$  charts based on different estimators as shown in Figure 3 indicate that there are no out-of-control points for the four control charts considered. The MAD-based control charts have narrower control limits, but the difference is not significant as such.

In Table 2, the CLI of dispersion charts based on different estimators for normal distribution are approximately the same. The  $Q_n$  chart has narrower CLI value followed by the  $D$  estimator chart. The  $S_n$ -based control charts have the smallest CLI, followed by the  $Q_n$ -based charts in Models 2 and 5, while the  $Q_n$ -based control charts have the smallest CLI in Models 3 and 4, followed by MAD and  $Q_n$ -based charts, respectively. The ARL, MRL, and SDRL also have the same results for the estimators. The results in Table 3 support the simulation results, the CLI of  $S_n$ -based chart has the smallest value, followed by that of  $Q_n$ -based chart for dispersion. In Figure 4,  $S_n$ - and  $Q_n$ -based control charts have approximately the same control limits, which indicates that they can detect any small shift or out-of-control faster than MAD and  $D$ -based control charts.

## 5 | CONCLUSION

This study investigates the performances of MAD- and AMAD-based control charts under normal, skewed, and heavy-tailed distributions. For all the distributions considered in this study, the CLI under the normal distribution differs significantly from the non-normal distributions. The result is not surprising, because the estimators considered are for non-normal data. However, there were no significant differences for all other performance indices used in this work. The results show that MAD and  $D$  are suitable estimators for standard deviation in constructing mean charts for skewed and heavy-tailed distributions. However, the MAD-based control chart detected out-of-control situations faster than  $S_n$ - and  $D$  estimator-based control charts. Thus, the  $S_n$  and  $Q_n$  are suitable estimators for standard deviation in constructing dispersion charts for skewed and heavy-tailed distributions. Therefore, the MAD-,  $S_n$ -, or  $D$ -based control charts can be used for the  $\bar{X}$  control charts, while  $S_n$ - and  $Q_n$ -based control charts are more appropriate for monitoring skewed and heavy-tailed distributions.

## DATA AVAILABILITY STATEMENT

The data used in this research is available from the Authors by request.

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## REFERENCES

1. Abbasi SA, Miller A. D chart: an efficient alternative to monitor process dispersion. Lecture notes in engineering and computer science. In: Proceedings of the World Congress on Engineering and Computer Science. Vol II. WCECS, San Francisco, USA; 2011:933-938.
2. Abu-Shawiesh MOA. A simple robust control chart based on MAD. *J Math Stat.* 2008;4(2):102-107.
3. Abu-Shawiesh MOA, Abdullah MB. Estimating the process standard deviation based on Downton estimator. *Qual Eng.* 2000;12(3):357-363.
4. Abu-Shawiesh MOA, Riaz M, Khaliq QU. MTSD-TCC: a robust alternative to Tukey's control chart based on the modified trimmed standard deviation. *J Math Stat.* 2020;8(3):262-277.
5. Adekeye KS, Azubuik PI. Derivation of the limits for control chart using the median absolute deviation for monitoring non normal process. *J Math Stat.* 2012;8(1):37-41.
6. Adekeye KS. Modified simple robust control chart based on median absolute deviation. *Int J Stat Prob.* 2012;1(2):91-95.
7. Adeoti OA, Olaomi JO, Adekeye KS. Control chart limits for monitoring process mean based on Downton's estimator. *Qual Reliab Eng Int.* 2016;32(5):1731-1740.
8. Aslam M, Srinivasa Rao G, AL-Marshadi AH, Ahmad L, Jun C. Control charts for monitoring process capability using MAD for some popular distributions. *Processes.* 2019;7:287.

9. Barnett FC, Mullen K, Saw JG. Linear estimates of a population scale parameter. *Biometrika*. 1967;54:551-554.
10. Celik N. Control charts based on robust scale estimators. *Am Res J Math*. 2015;1(1):41-48.
11. Das N. Control charts for controlling variability of non-normal process. *Econ Qual Control*. 2011;26:121-131.
12. Downton F. Linear estimates with polynomial coefficients. *Biometrika*. 1966;53:129-141.
13. Hampel FR. The influence curve and its role in robust estimation. *J Am Stat Assoc*. 1974;69:383-393.
14. Kenett R, Zacks S. *Modern Industrial Statistics: The Design and Control of Quality and Reliability*. Duxbury Press; 1998.
15. Koukouvinos C, Lappa A. A moving average control chart using a robust scale estimator for process dispersion. *Qual Reliab Eng Int*. 2019;35:2462-2493.
16. Langenberg P, Iglewicz B. Trimmed mean X and R charts. *J Qual Technol*. 1986;18:152-161.
17. Montgomery DC. *Introduction to Statistical Quality Control*. 6th ed. USA: John Wiley & Sons, Inc.; 2009.
18. Riaz M, Saghir A. Monitoring process variability using Gini's mean difference. *Qual Technol Quant Manag*. 2007;4(4):439-454.
19. Rocke DM. Robust control charts. *Technometrics*. 1989;31:173-184.
20. Rousseeuw PJ, Croux C. Alternatives to the median absolute deviation. *J Am Stat Assoc*. 1993;88:1273-1283.
21. Schoonhoven M, Does RJMM. A robust standard deviation control chart. *Technometrics*. 2012;54(1):73-82.
22. Shabbir A, Lin Z, Abbasi SA, Riaz M. On efficient monitoring of process dispersion using interquartile range. *Open J Appl Sci*. 2012;2(04):39-43.
23. Shahriari H, Maddahi A, Shokouhi AH. A robust dispersion control chart based on M-estimate. *J Ind Syst Eng*. 2009;2(4):297-307.
24. Schoonhoven M, Does RJMM. The X-bar control chart under non-normality. *Qual Reliab Eng Int*. 2010;26(2):167-176.

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