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ORIGINAL ARTICLE

Entropy generation analysis for variable viscous couple stress fluid flow through a channel with non-uniform wall temperature

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Abstract This article addresses the influence of couple stresses in minimizing of entropy generation rate associated with heat transfer irreversibility in the steady flow of a variable viscous fluid through a channel with a non-uniform wall temperature. The flow is induced by a constant axial pressure gradient applied in the flow direction. It is assumed that the fluid viscosity varies linearly with temperature. Analytical expressions for the dimensionless equations governing the fluid velocity and temperature are derived and used to obtain expressions for volumetric entropy generation numbers, irreversibility distribution ratio and the Bejan number in the flow field. Plots for different pertinent parameters entering the velocity and temperature fields are displayed and discussed.

1. Introduction

The major problem often encountered in energy generation is how to minimize or control energy wastages in form of heat dissipation. This has spurred a number of research works on the minimization of entropy generation especially when dealing with heat transfer problems. Of interest in this paper is the work done by Makinde [1] who gave a detailed thermodynamic analysis for a temperature dependent viscous fluid that is flowing steadily through a channel with non-uniform wall temperature. Interested readers can read the following papers for more interesting results on heat irreversibility in fluid flow based on second law to thermodynamics [2–20].

However, recent findings have shown that some fluids contain tiny polymer additives either to resist thermal effect as lubricating fluids in case of lubricants such as engine oil or to aid joint movement in the case of greasy synovial fluids. In human blood, tiny red blood cells are present for tissue respiration and many more real-life applications; therefore, a single constitutive equation cannot be used for non-Newtonian fluids. There a quite a lot of constitutive models are available in the literature to describe the rheological properties of these fluids but of interest in the present study is that introduced by Stokes' which takes into account the presence of couple stresses, body couples and non-symmetric stress tensor. This has been successfully used in the literature to describe fluid flow problems under different flow conditions [21–28] and references therein.

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Motivated by studies in [29–33] the objective of the present study was to improve mechanics of the thermal system described in [1] so as to accommodate a wide range of non-Newtonian fluids used in many industrial and engineering set-ups exchanging heat between two thermal reservoirs. The rest of the paper is organized as follows. Section 2 presents the mathematical formulation and non-dimensionalization of the problem. In Section 3, the method of solution is described while Section 4 deals with the discussion of results based on the physics of the problem. Finally, Section 5 contains final remarks.

2. Mathematical formulation

Consider the steady flow of an incompressible, viscous couple stress fluid flow through infinite parallel plates with non-uniform wall temperature. The flow is induced by an axial pressure gradient. Then the equations governing the hydrodynamically and thermodynamically fully developed fluid flow are the momentum and energy equations [1,30–33]:

$$0 = -\frac{dP}{dx} + \frac{d}{dy'}\left(\mu'\frac{du'}{dy'}\right) - \eta\frac{d^4u'}{dy'^4} \tag{1}$$

$$0 = \frac{d^2T}{dy'^2} + \frac{\mu'}{k}\left(\frac{du'}{dy'}\right)^2 + \frac{\eta}{k}\left(\frac{d^2u'}{dy'^2}\right)^2 \tag{2}$$

the appropriate conditions at the walls are

$$\left.\begin{array}{l} T(0) = T_0,\ T(h) = T_1 \\ u'(0) = \frac{d^2u'}{dy'^2}(0) = 0 = \frac{d^2u'}{dy'^2}(h) = u'(h) \end{array}\right\} \tag{3}$$

the linear variation of viscosity with temperature follows

$$\mu' = \mu_0(1 - \beta(T - T_0)) \tag{4}$$

Heat transfer to fluid flow with variable viscosity in a channel is irreversible. Hence, entropy production becomes continuous due to exchange of energy and momentum within the fluid particles in the channel. The expression for the total entropy generation within the fluid system can then be written as

$$E_G = \frac{k}{T_0^2}\left(\frac{dT}{dy'}\right)^2 + \frac{\mu'}{T_0}\left(\frac{du'}{dy'}\right)^2 + \frac{\eta}{T_0}\left(\frac{d^2u'}{dy'^2}\right)^2 \tag{5}$$

Introducing the following dimensionless variables and parameters

$$\begin{array}{l} y = \frac{y'}{h}, u = \frac{u'}{U}, \mu = \frac{\mu'}{\mu_0}, \theta = \frac{T-T_0}{T_1-T_0}, \alpha = \beta(T_1 - T_0), \\ \mathrm{Br} = \frac{\mu U^2}{k(T_1-T_0)},\ a^2 = \frac{\eta}{\mu h^2},\ N_S = \frac{T_0^2h^2E_G}{k(T_1-T_0)^2},\ \Omega = \frac{T_0}{T_1-T_0} \end{array} \tag{6}$$

we get

$$\frac{d^4u}{dy^4} = a^2G + a^2\frac{d}{dy}\left((1-\alpha\theta)\frac{du}{dy}\right) \tag{7}$$

$$\frac{d^2\theta}{dy^2} = -\mathrm{Br}\left\{(1-\alpha\theta)\left(\frac{du}{dy}\right)^2 + \frac{1}{a^2}\left(\frac{d^2u}{dy^2}\right)^2\right\} \tag{8}$$

$$N_S = \left(\frac{d\theta}{dy}\right)^2 + \frac{\mathrm{Br}}{\Omega}\left((1-\alpha\theta)\left(\frac{du}{dy}\right)^2 + \frac{1}{a^2}\left(\frac{d^2u}{dy^2}\right)^2\right) \tag{9}$$

the non-moving, stress-free upper wall of the channel is maintained at a given temperature

$$u(1) = \frac{d^2u}{dy^2}(1) = 0, \theta(1) = 1 \tag{10}$$

while the no-slip, stress-free conditions at the channel lower wall are also fixed and maintained at a temperature different from those of the upper wall,

$$u(0) = \frac{d^2u}{dy^2}(0) = 0 = \theta(0), \tag{11}$$

In Eqs. (1)–(11), y and y' – are dimensionless and dimensional distances measured in the normal direction respectively, h – is the channel width, and $(u,\ u'\ U)$ – are the dimensionless, dimensional and characteristic velocity respectively. $(\mu,\ \mu',\ \mu_0)$ – represents the dimensionless dynamic fluid viscosity, dimensional dynamic fluid viscosity and referenced dynamic fluid viscosity respectively. (θ, T, T_1, T_0) – are the dimensionless fluid temperature, dimensional fluid temperature, upper wall fluid temperature and lower wall fluid temperature respectively, P is the pressure, α and β – are the dimensionless and dimensional viscosity-variation parameters respectively, Br – is the Brinkman number, k – is the thermal conductivity a^2 – dimensionless couple stress inverse parameter, η – couple stress parameter, (E_G, N_S) are the dimensional and dimensionless entropy generation rate and Ω is the temperature difference parameter.

3. Method of solution

As suggested in [1], we assume that $0 < \alpha << 1$ and as such it is convenient to obtain a perturbative solution in the form:

$$\left.\begin{array}{l} \theta(y) = \sum_0^1 \theta_n(y)\alpha^n + O(\alpha)^2, \\ u(y) = \sum_0^1 u_n(y)\alpha^n + O(\alpha)^2. \end{array}\right\} \tag{10}$$

So that by substituting (10) in (7), (8) and equating coefficients, we get the following orders of

$$\begin{array}{l} O(\alpha)^0 : \frac{d^4u_0}{dy^4} = a^2\left(G + \frac{d^2u_0}{dy^2}\right); \\ \frac{d^2u_0}{dy^2}(0) = u_0(0) = 0 = \frac{d^2u_0}{dy^2}(1) = u_0(1), \\ O(\alpha)^1 : \frac{d^4u_1}{dy^4} = a^2\left(\frac{d^2u_1}{dy^2} - \frac{du_0}{dy}\frac{d\theta_0}{dy} - \theta_0\frac{d^2u_0}{dy^2}\right); \\ \frac{d^2u_1}{dy^2}(0) = u_1(1) = 0 = \frac{d^2u_1}{dy^2}(1) = u_1(1), \end{array} \tag{11}$$

together with

$$\begin{array}{l} O(\alpha)^0 : \frac{d^2\theta_0}{dy^2} = -\mathrm{Br}\left\{\left(\frac{du_0}{dy}\right)^2 + \frac{1}{a^2}\left(\frac{d^2u_0}{dy^2}\right)^2\right\}; \\ \theta_0(0) = 0, \theta_0(1) = 1, \\ O(\alpha)^1 : \frac{d^2\theta_1}{dy^2} = \mathrm{Br}\{\theta_0\left(\frac{du_0}{dy}\right)^2 - 2\frac{du_0}{dy}\frac{du_1}{dy} - \frac{2}{a^2}\frac{d^2u_0}{dy^2}\frac{d^2u_1}{dy^2}\}; \\ \theta_1(0) = 0, \theta_1(1) = 0. \end{array} \tag{12}$$

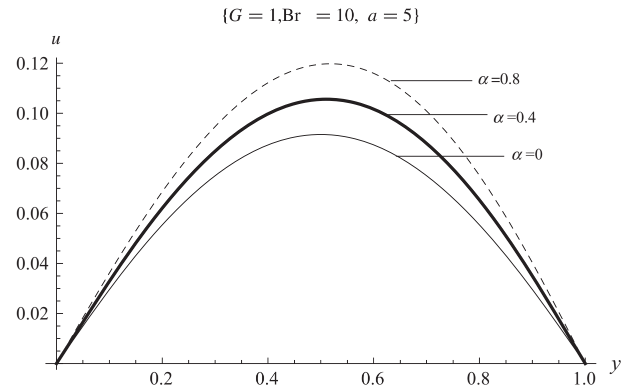

Figure 1 Effect of viscosity variation parameter on the velocity profile.

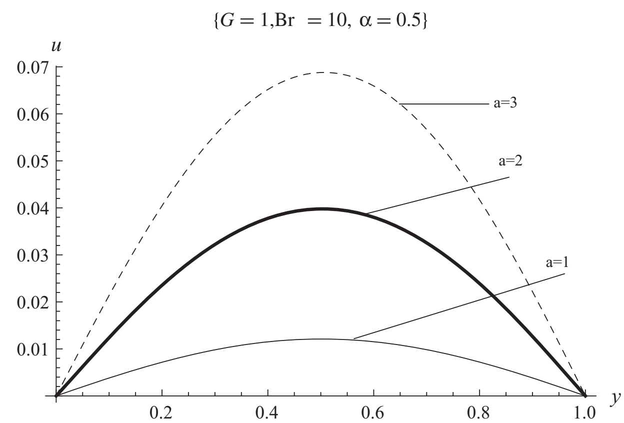

Figure 2 Effect of couple stress inverse parameter on the velocity profile.

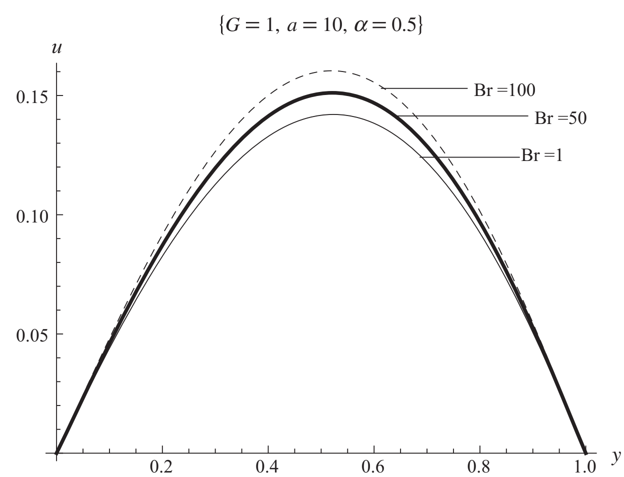

Figure 3 Effect of Brinkman number on the velocity profile.

Eqs. (11) and (12) are coded in the DSolve algorithm in a computer symbolic algebra package – MATHEMATICA to obtain the approximate solutions

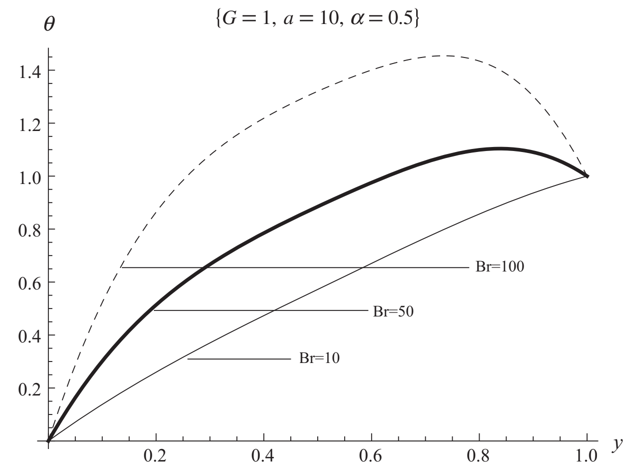

Figure 4 Effect of Brinkman number on the temperature profile.

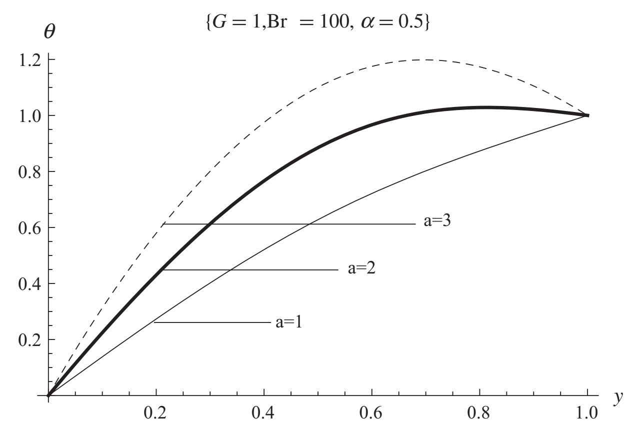

Figure 5 Effect of couple stress inverse parameter on the temperature profile.

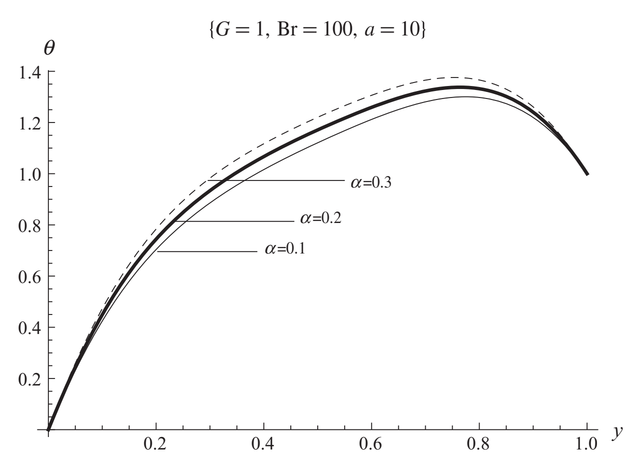

Figure 6 Effect of couple stress inverse on the temperature profile.

$$u(y) = \sum_{n=0}^{1} u_n \alpha^n, \quad \theta = \sum_{n=0}^{1} \theta_n \alpha^n. \tag{13}$$

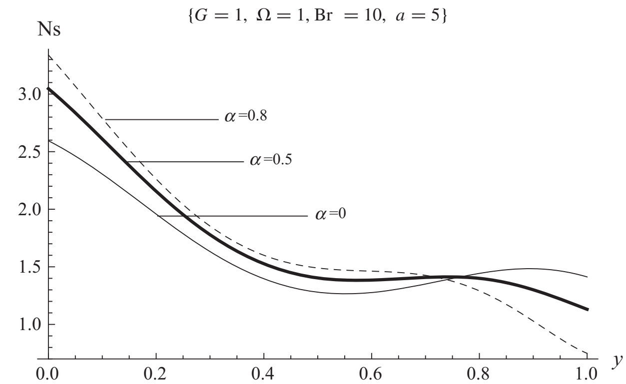

Figure 7 Effect of viscosity variation parameter on the temperature profile.

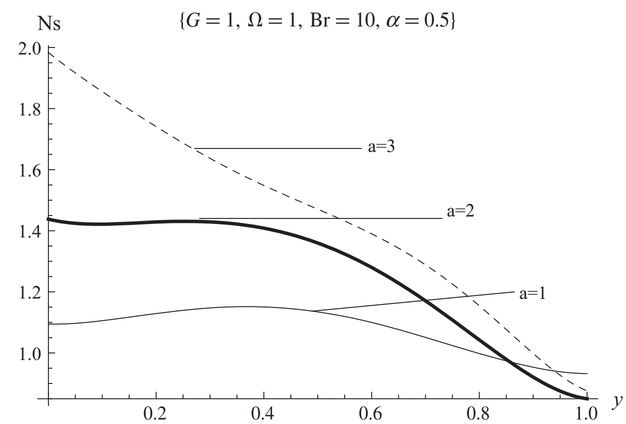

Figure 8 Effect of couple stress inverse on entropy generation rate.

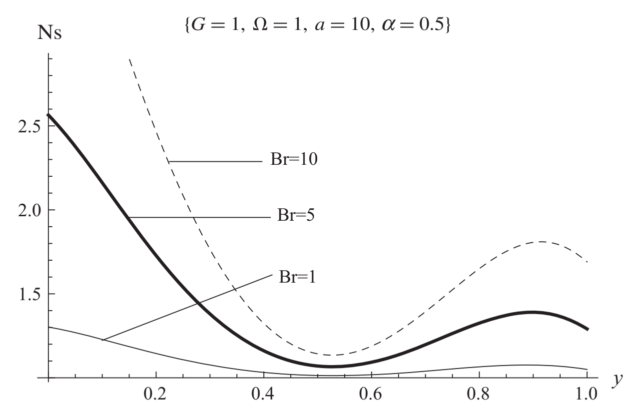

Figure 9 Effect of Brinkman number on entropy generation rate.

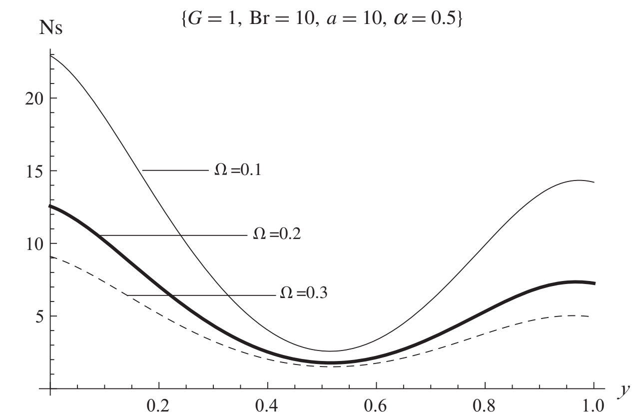

Figure 10 Effect of temperature difference parameter on entropy generation rate.

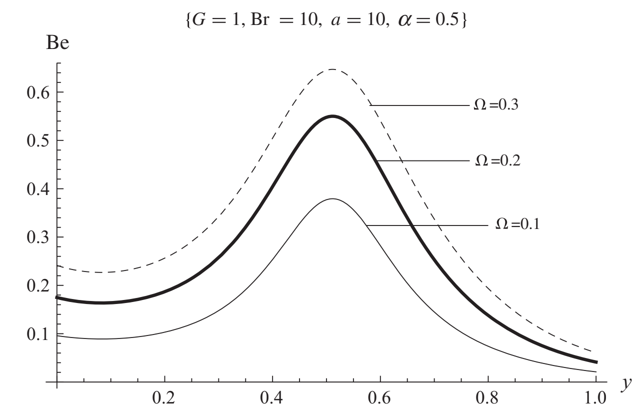

Figure 11 Effect of temperature difference parameter on irreversibility ratio.

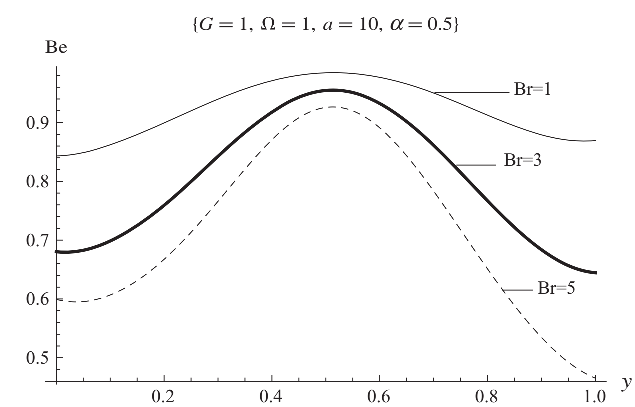

Figure 12 Effect of Brinkman number on irreversibility ratio.

Due to the huge size of the solutions obtained in (13), only the graphical solutions will be presented as in Figs. 1–6 in Section 4 below. The solutions (13) are substituted in (9) to obtain the entropy generation rate and the results are presented as in Figs. 7–10. If we divide the entropy generation N_S in two viz, N_1 represents the irreversibility due to heat transfer while N_2 represents the irreversibility due to fluid friction in the form

$$N_1 = \left(\frac{d\theta}{dy}\right)^2, \; N_2 = \frac{\mathrm{Br}}{\Omega}\left((1-\alpha\theta)\left(\frac{du}{dy}\right)^2 + \frac{1}{a^2}\left(\frac{d^2u}{dy^2}\right)^2\right). \tag{14}$$

Then the irreversibility ratio Be can be written as

$$Be = \frac{N_1}{N_1+N_2} = \frac{1}{1+\frac{N_2}{N_1}} = \frac{1}{1+\Phi}; \Phi = \frac{N_2}{N_1} \tag{15}$$

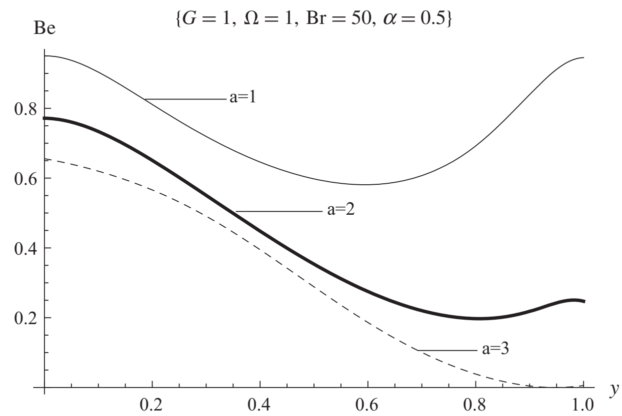

Figure 13 Effect of couple stress inverse on irreversibility ratio.

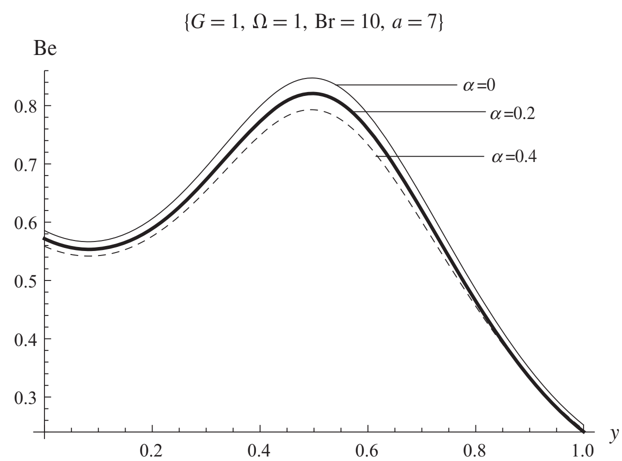

Figure 14 Effect of viscosity variation parameter on irreversibility ratio.

Eq. (15) is then used to obtain the irreversibility ration within the flow channel as presented in Figs. 11–14. It is evident from (15) that the numerical values lie between $0 \leqslant Be \leqslant 1$ depending on the values of Φ.

4. Results and discussion

To provide a physical insight into the synchronized flow problem, the numerical result of the analytical solutions is presented graphically and discussed. Fig. 1 shows the effect of viscosity variation parameter on the fluid flow velocity. It is observed from the graph that maximum velocity increases with an increase in the viscosity variation parameter. This is true since an increase in temperature implies a reduction in reluctance of the fluid to flow that is associated with the dynamic viscosity of the fluid. Hence the velocity of the fluid flow is expected to increase as shown in the graph. Fig. 2 shows the effect of couple stress inverse on the fluid flow. An increase in the couple stress inverse parameter physically means a decrease in the size of the additives to the fluid and decrease in the dynamic viscosity of the fluid. As a result, rise in the couple stress inverse parameter is expected to enhance the fluid flow. On the other hand the couple that is the inverse of a will eventually decrease the fluid flow velocity due to increased dynamic viscosity of the non-Newtonian fluid. Fig. 3 shows the effect of Brinkman number on the flow velocity. As observed, an increase in Brinkman number increases the fluid flow velocity. This is due to the conversion of heat energy to kinetic energy for the fluid particles within the channel.

Figs. 4–6 represent the temperature profile of the fluid. Fig. 4 represents the effect of Brinkman number on the temperature distribution within the channel. From the graph, it is observed that an increase in Brinkman number enhances the temperature distribution of the fluid due to viscous heating of the fluid particles within the flow channel. Similarly, an increase in the couple stress inverse is observed in Fig. 5 to increase the temperature distribution of the fluid. Invariably the rise in the couple stress parameter will decrease the fluid temperature as the non-Newtonian behavior improves. Fig. 6 depicts the effect of viscosity variation parameter on the fluid temperature distribution within the channel. From the graph it is observed that the fluid temperature increases with an increase in the viscosity variation parameter. This is true since an increase in the fluid temperature implies an increase in the motion of the fluid particles. The effective collision thus increases the fluid temperature distribution within the flow channel due to decrease in the fluid dynamic viscosity.

Fig. 7 represents the effect of viscosity variation parameter on the fluid flow. From the plot, it is observed that entropy generation rate decreases with an increase in the viscosity variation parameter in the fluid layer around the heated wall. This is so due to reduction in the heat transfer rate in the hot region; however, about the centerline of the channel there is rise in the entropy generation rate. Similar behavior is seen at the cold wall. This is physically true since heat is transferred from the heated wall through the non-Newtonian fluid to the cold wall. Thus rise in the viscosity variation parameter enhances the entropy generation rate due to additional heat generation from viscous heating of the fluid. More so, Fig. 8 represents the influence of couple stress inverse parameter on the entropy generation rate within the flow channel. As observed from the graph, an increase in the couple stress inverse parameter is observed to increase the entropy generation rate within the flow channel due to reduction in the dynamic fluid viscosity associated with the couple stress inverse. In reality, as the couple stress parameter increases fluid friction increases and this decreases the rate at which heat is transferred from the heated wall through the fluid thus minimizing the entropy production rate within the flow channel. Fig. 9 represents the influence of Brinkman number on the entropy generation rate. The result shows that entropy generation rate increases within the channel as Brinkman number increases. The reason for this is very clear, Brinkman number is a heat source and heat is generated within the layers of the moving fluid particles. The heat generated together with the heat transfer from the heat wall encourages entropy production within the flow channel. Therefore, to minimize entropy, Brinkman number needs to be controlled. Similar behavior is seen as the temperature difference at the walls increases in Fig. 10.

Finally, Figs. 11–14 show the irreversibility ratio for various parameter values. Fig. 11 represents the effect of temperature difference parameter on the irreversibility ratio. As shown in the graph, rise in the temperature difference parameter decreases the irreversibility due to viscous dissipation thus encouraging heat irreversibility due to heat transfer within the flow channel as seen in the plot. Fig. 12 illustrates the influence of Brinkman number on the heat irreversibility within the flow channel. As observed, as Brinkman number increases the fluid temperature distribution increases significantly due to viscous heating of the fluid. This implies that more and more heat is generated from the viscous heating of the fluid over the heat transfer from the heated wall to the fluid as shown in the graph. Thus heat irreversibility from viscous dissipation dominates over heat irreversibility due to heat transfer.

Fig. 13 represents the effect of couple stress inverse on heat irreversibility. As shown from the graph, an increase in the couple stress inverse (which represents a decrease in the dynamic fluid viscosity) is observed to enhance the dominance of fluid friction irreversibility over heat transfer. This is due to the fact that reduction in dynamic fluid viscosity will enhance fluid flow and as such heat will be generated due to fluid particle collision within the flow channel. In Fig. 14, fluid friction irreversibility dominates over irreversibility due to heat transfer as the viscosity variation parameter increases. This is true because rise in α implies a decrease in the temperature difference of the fluid. Thus the viscosity of the fluid is expected to increase which leads to rise in the viscous dissipation within the layer of the moving fluid.

5. Conclusion

In the present paper, we have examined the effect of couple stresses on the temperature dependent viscous flow through a channel with non-uniform wall temperature. The governing ordinary differential equations for momentum, energy and entropy generation are modeled, non-dimensionalized and solved analytically by using perturbation method. Effects of various parameters on velocity, temperature and entropy generation profiles are shown graphically. Summarily, we have showed that couple stress parameter has reducing effect on the flow velocity, temperature distribution within the flow channel and discourages entropy generation, therefore, conserving energy of the thermal system.

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